

Complete the following problems **without using a calculator** :

1. Evaluate:

$$a) \frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \boxed{\frac{1}{6}}$$

$$b) \frac{1}{2} \div \left(\frac{-1}{4}\right) = \frac{1}{2} \times \left(-\frac{4}{1}\right) = \boxed{-2}$$

$$c) 7(3^2 - 4) = 7(9 - 4) = 7(5) = \boxed{35}$$

$$d) 2^{-3} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$$

$$e) (3^2)^{-1} = 3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$$

2. Evaluate and express your answer as a decimal:

$$a) 5 + \frac{3}{2} = \frac{10}{2} + \frac{3}{2} = \frac{13}{2} = 6\frac{1}{2} = \boxed{6.5}$$

$$b) \frac{3}{4} - 2 = \frac{3}{4} - \frac{8}{4} = -\frac{5}{4} = -1\frac{1}{4} = \boxed{-1.25}$$

3. Simplify:

$$a) 8^{\frac{2}{3}} 2^{-\frac{1}{3}} = (2^3)^{\frac{2}{3}} 2^{-\frac{1}{3}} = 2^2 2^{-\frac{1}{3}} = 2^{2-\frac{1}{3}} = 2^{\frac{6}{3}-\frac{1}{3}} = \boxed{2^{\frac{5}{3}}}$$

$$b) \sqrt{16x^6} = \sqrt{4^2(x^3)^2} = \boxed{4x^3}$$

$$c) 10^{\frac{1}{2}} 10^{\frac{3}{2}} = 10^{\frac{1}{2}+\frac{3}{2}} = 10^{4/2} = 10^2 = \boxed{100}$$

4. Simplify:

$$a) 10x - (3x - (7x + 2)) = 10x - (3x - 7x - 2) = 10x - (-4x - 2) \\ = 10x + 4x + 2 = \boxed{14x + 2}$$

$$b) \left(\frac{4/5x}{-2/15x}\right) = \frac{4}{5x} \times \frac{-15x}{2} = (-2)(-3) = \boxed{-6}$$

5. Find the product:

$$a) (4x+6)^2 = (4x+6)(4x+6) = 16x^2 + 24x + 24x + 36 \\ = \boxed{16x^2 + 48x + 36}$$

$$b) (2x+1)(5x-8) = 10x^2 + 5x - 16x - 8 = \boxed{10x^2 - 11x - 8}$$

$$c) 6x(3x^2 - 7x + 2) = \boxed{18x^3 - 42x^2 + 12x}$$

6. (Please write the fractions in the simplest form.)

a) Convert 35% to a fraction. Convert 35% to a decimal.

$$35\% = \frac{35}{100} = \frac{7}{20}$$

$$35\% = 0.35$$

b) Convert 0.12 to a percent. Convert 0.12 to a fraction.

$$0.12 = 12\%$$

$$0.12 = \frac{12}{100} = \frac{3}{25}$$

c) Convert  $\frac{3}{5}$  to a percent. Convert  $\frac{3}{5}$  to a decimal.

$$\frac{3}{5} = \frac{x}{100}$$

$$3(100) = 5x$$

$$\frac{300}{5} = x$$

$$x = 60$$

since  $\frac{3}{5} = \frac{60}{100}$  then

$$\frac{3}{5} = 60\% = 0.6$$

7. Solve for  $x$ .

a)  $12x - 8 = -2$

$$12x = -2 + 8$$

$$12x = 6$$

$$x = 6/12 = \frac{1}{2}$$

b)  $\frac{2x+5}{10x+3} = \frac{1}{3}$

$$(2x+5)3 = (10x+3)1$$

$$6x+15 = 10x+3$$

$$15-3 = 10x-6x$$

$$12 = 4x$$

$$\frac{12}{4} = x$$

$$3 = x$$

8. a) What is  $\frac{1}{3}$  of  $\frac{3}{5}$ ?  $(\frac{1}{3}) \times (\frac{3}{5}) = \boxed{\frac{1}{5}}$

b) What is 20% of 84?  $\begin{array}{r} 84 \\ \times .20 \\ \hline 16.80 \end{array}$   $\boxed{16.80}$

c) 75% of what number is 36?  $75\% = \frac{3}{4}$   $\frac{3}{4}x = 36$   
 $x = \frac{36}{\frac{3}{4}} = 36(4/3)$   
 $\boxed{x = 48}$

9. Your average so far in a class is 88. This average counts for 75% of your grade and the final exam counts for 25% of your grade. What do you need to make on the final exam for your course average to be at least 90?

$$\begin{array}{r} 88 \\ \times 0.75 \\ \hline 440 \\ 616 \\ \hline 66.00 \end{array}$$

$$88(0.75) + x(0.25) = 90$$

$$66 + x(0.25) = 90$$

$$x(0.25) = 24$$

$$x(\frac{1}{4}) = 24$$

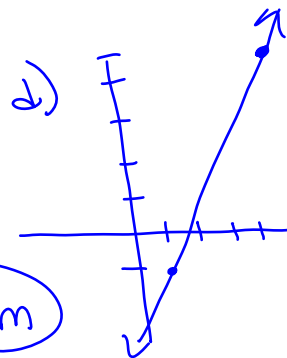
$$x = (24)(4)$$

$$\boxed{x = 96}$$

10. A line contains the points (4,5) and (1, -1).

- Find the equation of the line.
- What is the slope of the line?
- What is the y - intercept of the line?
- Graph the line.

$$\text{SLOPE} = \frac{\text{change in } y}{\text{change in } x} = \frac{5 - (-1)}{4 - 1} = \frac{6}{3} = 2 = m$$



$y = mx + b$  where  $m = \text{slope}$  and  $b = y$  intercept  
 to find the  $y$  intercept, substitute  $m = 2$  and either given point and solve for  $b$ .

$$y = mx + b$$

$$5 = 2(4) + b$$

$$5 = 8 + b$$

$$\boxed{-3 = b}$$

$$\text{So: } \begin{array}{l} \text{a) } y = 2x - 3 \\ \text{b) } m = 2 \\ \text{c) } b = -3 \end{array}$$

11. Consider the equation  $y = 3x - 8$

a) Find  $y$  when  $x = -1$ .

$$y = 3(-1) - 8 = -3 - 8 = \boxed{-11}$$

b) Find  $x$  when  $y = 2$ .

$$2 = 3x - 8$$

$$10 = 3x$$

$$\boxed{\frac{10}{3} = x}$$

12. a) Write using scientific notation: 0.0000683

$$0.0000683 = \boxed{6.83 \times 10^{-5}}$$

b) Write in decimal form without scientific notation:  $1.82 \times 10^{-6}$

$$\boxed{0.00000182}$$

13. Evaluate:

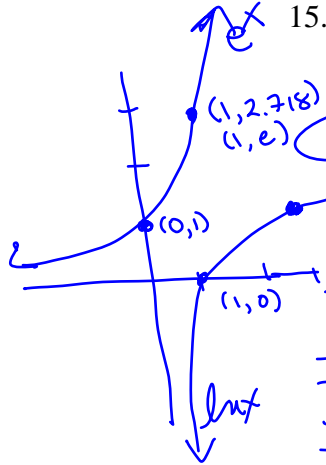
a)  $\log_2 8 = x \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow \boxed{x = 3}$

b)  $\ln \sqrt{e} = \ln(e^{1/2}) = \boxed{1/2}$

c)  $e^{2 \ln 2} = e^{\ln 2^2} = 2^2 = \boxed{4}$

14. Simplify:  $\ln 3x - \ln 9x = \ln(3x) + \ln(9x)^{-1}$   
 $= \ln\left(\frac{3x}{9x}\right) = \boxed{\ln\left(\frac{1}{3}\right)}$

15. Select the best answer:



- I.  $e \approx 2.718$     II.  $e \approx 3.14$     III.  $e = \ln(1)$

- a) I only is true  
 b) II only is true  
 c) III only is true  
 d) I and III are true

I.  $e \approx 2.718$  is true.  $e$  is a constant approx  $\approx 2.718$   
 II.  $\pi \approx 3.14$  (not  $e$ ) so this statement is false  
 III.  $e \approx 2.718$ ,  $\ln(1) = 0$  (since  $e^0 = 1$ ) so this statement is false.

16. The price of gas increased by 8% in June and another 15% in July. What was the total percentage increase from June 1 to July 31?

$$\begin{array}{r}
 1.08 \\
 \times 1.15 \\
 \hline
 1540 \\
 108 \\
 \hline
 1.2420
 \end{array}$$

1.242  $\Rightarrow$   
 $24.2\%$   
 increase for  
 the two months

$$\rightarrow P(\text{nausea}) = 0.3 \quad P(\text{good}) = 0.7$$

17. Approximately 30% of patients receiving a medication experience nausea.
- Suppose 2 patients are selected at random. Find the probability that both patients experience nausea.
  - Suppose 2 patients are selected at random. Find the probability that neither experience nausea.
  - Suppose 2 patients are selected at random. Find the probability that exactly one experiences nausea.
  - Suppose 2 patients are selected at random. Find the probability that at least one experiences nausea.

EACH OF THE TWO PATIENTS COULD BE EITHER "NAUSEOUS" OR "GOOD". SO THE POSSIBLE EVENTS ARE:  $\{(N,N), (N,G), (G,N), (G,G)\}$  } ← calculate these individual probabilities first

$$P(\text{BOTH ARE NAUSEOUS}) = P(N,N) = P(\text{1st NAUSEOUS AND 2nd NAUSEOUS}) = P(N)P(N) = (0.3)(0.3) = \underline{0.09}$$

$$P(\text{1st NAUSEOUS AND 2nd GOOD}) = P(N,G) = P(N)P(G) = (0.3)(0.7) = \underline{0.21}$$

$$P(\text{1st GOOD AND 2nd NAUSEOUS}) = P(G,N) = P(G)P(N) = (0.7)(0.3) = \underline{0.21}$$

$$P(\text{1st GOOD AND 2nd GOOD}) = P(G,G) = P(G)P(G) = (0.7)(0.7) = \underline{0.49}$$

$$\text{CHECK } 0.09 + 0.21 + 0.21 + 0.49 = 1.00 \checkmark$$

$$\text{SO, a) } P(\text{BOTH ARE NAUSEOUS}) = \boxed{0.09}$$

$$\text{b) } P(\text{NEITHER NAUSEOUS}) = \boxed{0.49}$$

$$\text{c) } P(\text{EXACTLY ONE IS NAUSEOUS}) = P(N,G) \text{ OR } (G,N) = P(N,G) + P(G,N) = 0.21 + 0.21 = \boxed{0.42}$$

$$\text{d) } P(\text{AT LEAST ONE IS NAUSEOUS}) = P(N,G) \text{ OR } (G,N) \text{ OR } (N,N) = P(N,G) + P(G,N) + P(N,N) = 0.21 + 0.21 + 0.09 = \boxed{0.51}$$