Complete the following problems **without using a calculator**:

1. Evaluate:
   a) $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
   b) $\frac{1}{2} \div \left(-\frac{1}{4}\right) = \frac{1}{2} \times (-\frac{4}{1}) = -2$
   c) $7(3^2 - 4) = 7(9 - 4) = 7(5) = 35$
   d) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
   e) $(3^2)^{-1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

2. Evaluate and express your answer as a decimal:
   a) $5 + \frac{3}{2} = \frac{10}{2} + \frac{3}{2} = \frac{13}{2} = 6\frac{1}{2} = 6.5$
   b) $\frac{3}{4} - 2 = \frac{3}{4} - \frac{8}{4} = -\frac{5}{4} = -1\frac{1}{4} = -1.25$
3. Simplify:
   a) \[ \left(2^3\right)^\frac{2}{3} = 2^{3 \cdot \frac{2}{3}} = 2^2 = 4 \]
   b) \[ \sqrt[3]{16x^6} = \sqrt[3]{4^2(x^3)^2} = 4x^2 \]
   c) \[ 10^3 \cdot 10^2 = 10^{3+2} = 10^5 = 100,000 \]

4. Simplify:
   a) \[ 10x - (3x - (7x + 2)) = 10x - (3x - 7x - 2) = 10x - (-4x - 2) = 14x + 2 \]
   b) \[ \frac{\left(\frac{4}{5}x\right)}{\left(\frac{-2}{15x}\right)} = \frac{4x}{5x} \cdot \frac{15x}{-2} = \frac{2x}{-1} = -2x \]

5. Find the product:
   a) \[ (4x+6)^2 = (4x+6)(4x+6) = 16x^2 + 24x + 24x + 36 = 16x^2 + 48x + 36 \]
   b) \[ (2x+1)(5x-8) = 10x^2 + 5x - 16x - 8 = 10x^2 - 11x - 8 \]
   c) \[ 6x(3x^2 - 7x + 2) = 18x^3 - 42x^2 + 12x \]
6. (Please write the fractions in the simplest form.)
   a) Convert 35% to a fraction. Convert 35% to a decimal.
      \[ 35\% = \frac{35}{100} = \frac{7}{20} \]
      \[ 35\% = 0.35 \]
   b) Convert 0.12 to a percent. Convert 0.12 to a fraction.
      \[ 0.12 = 12\% \]
      \[ 0.12 = \frac{12}{100} = \frac{3}{25} \]
   c) Convert \( \frac{3}{5} \) to a percent. Convert \( \frac{3}{5} \) to a decimal.
      \[ \frac{3}{5} = \frac{x}{100} \]
      \[ 3 \times 100 = 5x \]
      \[ \frac{300}{5} = x \]
      \[ x = \frac{60}{100} \text{ then } \frac{3}{5} = \frac{60\%}{0.6} \]

7. Solve for \( x \).
   a) \( 12x - 8 = -2 \)
      \[ 12x = -2 + 8 \]
      \[ 12x = 6 \]
      \[ x = \frac{1}{2} = \boxed{\frac{1}{2}} \]
   b) \( \frac{2x + 5}{10x + 3} = \frac{1}{3} \)
      \[ (2x + 5) 3 = (10x + 3) 1 \]
      \[ 6x + 15 = 10x + 3 \]
      \[ 15 - 3 = 10x - 6x \]
      \[ 12 = 4x \]
      \[ \frac{12}{4} = x \]
      \[ \boxed{3} = x \]
8. a) What is $\frac{1}{3}$ of $\frac{3}{5}$? 
\[ \left( \frac{1}{3} \right) \times \left( \frac{3}{5} \right) = \frac{1}{5} \]

b) What is 20% of 84?
\[ \begin{array}{c}
\times \quad \times \\
84 \quad 20 \\
\hline
16.80 \\
\end{array} \]

9. 75% of what number is 36?
\[ \frac{3}{4} \times x = 36 \\
\frac{3}{4}x = 36 \\
x = \frac{36 \times 4}{3} \\
x = 48 \]

9. Your average so far in a class is 88. This average counts for 75% of your grade and the final exam counts for 25% of your grade. What do you need to make on the final exam for your course average to be at least 90?
\[ \begin{array}{c}
88 \quad 0.75 \\
0.75 \times 88 = 66 \\
66 + x \times 0.25 = 90 \\
66 + 0.25x = 90 \\
0.25x = 24 \\
x = \frac{24}{0.25} \\
x = 96 \\
\end{array} \]

10. A line contains the points (4,5) and (1,-1).
a) Find the equation of the line.
b) What is the slope of the line?
c) What is the y – intercept of the line?
d) Graph the line.
\[ \text{slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{5 - (-1)}{4 - 1} = \frac{6}{3} = 2 \]
\[ y = mx + b \quad \text{where} \quad m = \text{slope and} \quad b = \text{y intercept} \]
\[ \text{to find the y intercept, substitute} \quad m = 2 \quad \text{and} \quad \text{either given point and solve for} \quad b. \]
\[ y = 2x + b \]
\[ 5 = 2(4) + b \\
5 = 8 + b \\
-3 = b \]
11. Consider the equation  $y = 3x - 8$
   a) Find $y$ when $x = -1$.  
      \[ y = 3(-1) - 8 = -3 - 8 = -11 \]
   b) Find $x$ when $y = 2$.  
      $2 = 3x - 8$
      \[ 10 = 3x \]
      \[ \frac{10}{3} = x \]

12. a) Write using scientific notation:  $0.0000683 = 6.83 \times 10^{-5}$
   b) Write in decimal form without scientific notation:  $1.82 \times 10^{-6}$

13. Evaluate:
   a) $\log_2 8 = x \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$
   b) $\ln \sqrt{e} = \ln (e^{1/2}) = \frac{1}{2}$
   c) $e^{2 \ln 2} = e^{\ln 2^2} = 2^2 = 4$

14. Simplify:  $\ln 3x - \ln 9x = \ln (\frac{3x}{9x}) = \ln (\frac{1}{3})$
15. Select the best answer:

I. $e \approx 2.718$  
II. $e \approx 3.14$  
III. $e = \ln(1)$

a) I only is true  
b) II only is true  
c) III only is true  
d) I and III are true

I. $e \approx 2.718$ is true. $e$ is a constant approx $\approx 2.718$

II. $\pi \approx 3.14$ (not $e$) so this statement is false

III. $e \approx 2.718$, $\ln(1) = 0$ (since $e^0 = 1$) so this statement is false.

16. The price of gas increased by 8% in June and another 15% in July. What was the total percentage increase from June 1 to July 31?

\[
\begin{array}{c}
1.08 \\
\times 1.15 \\
\hline
1.242 = \Rightarrow \\
\end{array}
\]

Increase for the two months

24.2%
17. Approximately 30% of patients receiving a medication experience nausea.
   
a) Suppose 2 patients are selected at random. Find the probability that both patients experience nausea.
   
b) Suppose 2 patients are selected at random. Find the probability that neither experience nausea.
   
c) Suppose 2 patients are selected at random. Find the probability that exactly one experiences nausea.
   
d) Suppose 2 patients are selected at random. Find the probability that at least one experiences nausea.

\[ P(\text{nausea}) = 0.3 \quad P(\text{good}) = 0.7 \]

Each of the two patients could be either "nauseous" or "good." So the possible events are: \( \{(N,N), (N,G), (G,N), (G,G)\} \).

\[ P(\text{both are nauseous}) = P(N,N) = P(\text{1st nauseous and 2nd nauseous}) = P(N)P(N) = (0.3)(0.3) = 0.09 \]

\[ P(\text{1st nauseous and 2nd good}) = P(N,G) = P(N)P(G) = (0.3)(0.7) = 0.21 \]

\[ P(\text{1st good and 2nd nauseous}) = P(G,N) = P(G)P(N) = (0.7)(0.3) = 0.21 \]

\[ P(\text{1st good and 2nd good}) = P(G,G) = P(G)P(G) = (0.7)(0.7) = 0.49 \]

Check: 0.09 + 0.21 + 0.21 + 0.49 = 1.00

So a) \[ P(\text{both are nauseous}) = \boxed{0.09} \]

b) \[ P(\text{neither nauseous}) = \boxed{0.49} \]

c) \[ P(\text{exactly one is nauseous}) = P(N,G) \text{ or } (G,N) \]
   \[ = P(N,G) + P(G,N) = 0.21 + 0.21 = \boxed{0.42} \]

d) \[ P(\text{at least one is nauseous}) = \]
   \[ = P(\text{N,G}) \text{ or } (G,N) \text{ or } (N,N) \]
   \[ = P(N,G) + P(G,N) + P(N,N) = 0.21 + 0.21 + 0.09 = \boxed{0.51} \]